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## RESEARCH MEMORANDUM

SUMMARY OF SOME EFFECTIVE AERODYNAMIC TWISTING-MOMENT  
COEFFICIENTS OF VARIOUS WING-CONTROL CONFIGURATIONS  
AT MACH NUMBERS FROM 0.6 TO 1.7 AS DETERMINED  
FROM ROCKET-POWERED MODELS

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NATIONAL ADVISORY COMMITTEE  
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## NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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## SUMMARY

A summary is presented of some effective aerodynamic twisting-moment coefficients of various wing-control configurations for use at Mach numbers from 0.6 to 1.7 as determined by the Langley Pilotless Aircraft Research Division by the use of rocket-propelled models. The values thus obtained were determined by the combined use of the experimentally determined free-flight data and subsonic aerodynamic theory.

The results indicate that, within the framework of the necessary assumptions, the value of the effective twisting-moment coefficient decreases as the sweepback of the aileron hinge axis is increased.

Large changes in the value of the effective twisting-moment coefficient were obtained in the Mach number region from  $M \approx 0.8$  to  $M \approx 1.2$  with changes in aileron span and location upon the same wing plan form. Above  $M \approx 1.2$  the values tended to agree much more closely. This factor limits the use of these data to wing-control configurations similar to those tested.

Comparative tests of an outboard 0.3-span, 0.25-chord aileron and a midspan spoiler of approximately the same span length indicate that the twisting moment of the spoiler is about one-third that of the aileron for equal values of rolling effectiveness.

## INTRODUCTION

The problem of determining the proper aircraft structural stiffness is the problem of achieving the maximum stiffness for the minimum weight.

One particular phase of this problem is the determination of the effects of wing twisting upon lateral control effectiveness.

A method for evaluating the wing stiffness requirements for specified lateral control effectiveness for unswept wings at subsonic speeds below the critical Mach number is described in reference 1. This method makes use of the section twisting-moment coefficient for constant lift, the wing torsional stiffness at the mid-aileron location, and a nondimensional aeroelastic weighing factor to determine the loss in rolling effectiveness due to wing twisting. This method, while very successful for unswept wings at subsonic speeds, is not directly applicable at the present time to swept wings at transonic and supersonic speeds because of the lack of information regarding the section twisting moment in these speed ranges and the questionable merit of using values of the aeroelastic weighing factor which were derived based upon subsonic lifting-line theory for swept wings at transonic and supersonic speeds.

In order to circumvent these difficulties, the technique of reference 2 was initiated whereby experimentally determined values of the loss in control effectiveness due to wing twisting are used in an adaptation of reference 1 to obtain effective aerodynamic twisting moments for some straight and swept wings for Mach numbers from 0.6 to 1.7 which can be used to estimate the loss in control effectiveness throughout the transonic region and up to a Mach number of 1.7. Because these values of the effective twisting-moment coefficient are based upon subsonic aerodynamic theory, the use of these values should be restricted to wing-control configurations very similar to those for which data are available.

The purpose of this investigation is to summarize the effective twisting-moment coefficients recently obtained for various wing-control configurations and previously reported in references 2 to 5. All the data in the present paper were calculated using new values of the aeroelastic weighing factor which were based upon subsonic lifting-surface theory which should be more realistic for swept wings and wings of low aspect ratio than lifting-line theory. All the data were obtained in free flight from rocket-propelled test vehicles which permits evaluation of the rolling power of wing-control configurations continuously over a Mach number range of approximately 0.6 to 1.7.

#### SYMBOLS

- A aspect ratio ( $b^2/s$ )
- b diameter of circle swept by wing tips (with regard to rolling characteristics, this diameter is considered to be effective span of three-fin models), feet

|                                      |  |
|--------------------------------------|--|
| c                                    | wing chord, parallel to free stream, feet  |
| EI                                   | bending stiffness in planes perpendicular to 40-percent-chord line, pound-inches <sup>2</sup>  |
| GJ                                   | torsional stiffness in planes perpendicular to 40-percent-chord line, pound-inches <sup>2</sup>  |
| h/c                                  | spoiler height, fraction of local free-stream chord  |
| S                                    | area of two wings measured to fuselage center line, square feet  |
| m                                    | concentrated couple, applied near wing tip in a plane parallel to free stream and normal to wing-chord plane, foot-pounds                |
| q                                    | dynamic pressure, pounds per square foot   |
| p                                    | rolling velocity, radians per second   |
| V                                    | flight-path velocity, feet per second  |
| pb/2V                                | wing-tip helix angle, radians  |
| $c_{m\delta}$                        | rate of change of section pitching-moment coefficient with aileron angle at constant lift, per radian                                    |
| $\alpha_\delta$                      | rate of change of wing angle of attack with aileron angle as obtained for constant lift at section                                       |
| $\frac{c_{m/\delta}}{\alpha/\delta}$ | effective section twisting-moment coefficient for constant lift  |
| K                                    | nondimensional semispan station $\left(\frac{y}{b/2}\right)$   |
| $\lambda$                            | ratio of tip chord to root chord at model center line  |
| $\Lambda$                            | angle of sweep of quarter-chord line, degrees  |
| $\theta$                             | angle of twist, produced by m, at any section along wing span in a plane parallel to free stream and normal to wing-chord plane, radians |
| $\theta/m$                           | wing-torsional-stiffness parameter, measured parallel to model center line, radians per foot-pound                                       |
| $\tau$                               | derived aeroelastic weighing factor  |

$\phi$  fraction of rigid-wing rolling effectiveness retained by flexible wing

Subscripts:

a aileron  
i inboard end of aileron  
o outboard end of aileron  
r reference station (middle of exposed control span)

### MODELS AND TECHNIQUE

A typical test vehicle of the type used in this investigation is illustrated in the photograph presented as figure 1 and in the sketch of figure 2. The test wings are described in table I. Several test models of different degrees of wing stiffness were flown for each wing-control configuration in order to determine the loss in rolling effectiveness due to aeroelasticity. The results from these individual flights as well as complete descriptions of the various types of construction used in the individual models were previously reported in references 2 to 5. The models had approximately zero yaw and pitch.

The flight tests were made at the Langley Pilotless Aircraft Research Station at Wallops Island, Va. The test vehicles were propelled by a two-stage rocket-propulsion system up to a Mach number of about 1.7. During a period of approximately 10 seconds of coasting flight following the sustainer rocket-motor burnout, time histories of the rolling velocity were obtained with special radio equipment, the flight-path velocity was obtained by the use of CW Doppler radar, and the space coordinates were obtained by means of SCR 584 radar. These data, in conjunction with atmospheric data obtained with radiosondes, permit evaluation of the rolling effectiveness in terms of the parameter  $pb/2V$  as a function of Mach number. The Reynolds number for the tests varied from approximately  $3 \times 10^6$  at  $M = 0.6$  to  $7 \times 10^6$  at  $M = 1.7$  (based on mean wing chord).

### RESULTS AND DISCUSSION

#### Résumé and Background

Normally, the problem is to determine the stiffness that a given wing must have in order to prevent the rolling effectiveness from going

below a specified value, or, if the stiffness is specified, to determine the loss in rolling effectiveness to be expected. Either of these cases presupposes that the magnitude of the twisting-moment coefficient  $\frac{dc_m/d\delta}{d\alpha/d\delta}$  is known. However, section twisting-moment measurements in the transonic and supersonic speed ranges have not been available and this has prevented the use of this technique at these speeds. To overcome this lack of data, a method was presented in reference 2 whereby the experimentally determined loss in rolling effectiveness as obtained by rocket-powered models can be used to determine effective values of the section twisting-moment coefficient throughout a Mach number range of approximately 0.7 to 1.6. The values of the effective aerodynamic twisting-moment coefficients  $\frac{dc_m/d\delta}{d\alpha/d\delta}$  (hereafter referred to as  $c_{m\delta}/\alpha_\delta$  for simplicity) presented were determined by the use of equation (1) of reference 1 which is presented in a more useful form as follows:

$$\frac{c_{m\delta}}{\alpha_\delta} = \frac{2A^2}{\tau b^3 q} \frac{1 - \phi}{(\theta/m)_r} \quad (1)$$

For any given wing of aspect ratio  $A$  and span  $b$ , the torsional stiffness parameter  $(\theta/m)_r$  can be obtained by direct measurement. The value  $\tau$ , which can be calculated, is a nondimensional weighing factor which takes into account the spanwise variation of torsional stiffness, the plan form of the wing, and the location of the aileron upon the wing and which is also proportional to the loss in rolling effectiveness per unit twist at the reference station. The loss in rolling effectiveness  $(1 - \phi)$  at a given dynamic pressure  $q$  can be obtained from experimental data. By substituting these quantities into the basic equation, the only unknown factor remaining is the section twisting-moment coefficient for which the equation can then be solved.

#### Torsional Stiffness Parameter, $(\theta/m)_r$

The torsional stiffness parameters of all the test wings were obtained by applying a known couple near the wing tip and measuring the resulting twist along the span. It may be of interest to note that the wing torsional stiffness was obtained in the same manner for both straight and swept wings. That is, the couple was applied and the twist was measured in planes parallel to the model center line and normal to the wing-chord plane. The reason for measuring the torsional stiffness in this manner is based upon previous work (reference 6) which indicated that the steady-state rolling effectiveness  $(pb/2V)$  due to differential wing

incidence is independent of the angle of sweepback when the angle of incidence is measured in the direction of flight.

#### Loss in Rolling Effectiveness, $(1 - \phi)$

The loss in rolling effectiveness due to wing twist should also be independent of wing sweep if the twist is measured in the direction of flight. Therefore, only the moments which cause angular changes in the direction of flight need be considered. From these considerations, for steady-state rolling effectiveness, it is possible to conclude that the ratio of wing flexural stiffness to wing torsional stiffness ( $EI/GJ$ ), normally of great importance, should be relatively unimportant providing that the wing torsional stiffness is determined in the manner previously described. This is borne out by the fact that, within the experimental accuracy, a cross plot of  $pb/2V$ , the rolling effectiveness parameter, against  $(\theta/m)_r$ , the torsional stiffness parameter, is linear although the ratio of the stiffness in torsion to the stiffness in bending for the individual models may vary by as much as 400 percent. This is illustrated in figure 3 which presents some typical examples. In figure 3(c) the very large amount of wing sweep incorporated in this plan form ( $61^\circ$ ) normally makes the determination of the loss due to flexibility much more difficult and demonstrates the relative simplicity of this method. The linearity of the variation of  $pb/2V$  with  $(\theta/m)_r$  also indicates that the effects of wing bending due to the differences in wing loading which exist at steady-state roll are also relatively unimportant. Therefore, the loss in the steady-state rolling effectiveness due to wing flexibility for a given wing-control configuration is due primarily to the twisting moment in the direction of flight caused by the control and corresponds to previous experience with unswept wings. It should therefore be possible to utilize the observed losses in rolling performance and the known structural properties of the test wings to determine the aerodynamic twisting moments upon which the losses depend.

#### Theoretical Nondimensional Aeroelastic Weighing Factor, $\tau$

The effective twisting-moment coefficients presented in references 2 and 3 were obtained using values of  $\tau$  presented in reference 1. These values of  $\tau$  are strictly applicable only to unswept wings of aspect ratio 5 or larger owing to the method of derivation which was based on lifting-line theory (reference 7). However, new values of  $\tau$  have been computed using the equations presented in reference 1 and based upon lifting-surface theory (reference 8) in order to include wings of low aspect ratio and high sweep, linearly tapered wings varying from taper ratios of 0 to 1.0, and for controls of any spanwise location. These new values are presented in tabular form (table II) to permit easy, accurate



estimation of  $\tau$  for aspect ratios of 2 to 8. In order to illustrate the very large changes in the values of  $\tau$  which accompany movement of an aileron of given span inboard, a typical set of values from table II is presented in figure 4. All the values of the effective twisting-moment coefficient  $c_{m\delta}/\alpha_\delta$ , presented in this paper, were computed using the new values of  $\tau$  given in table II.

### Presentation of Results

The data presented in figure 5 were obtained by using the experimentally determined values of  $(1 - \phi)$  and the  $\tau$  values from table II. The data show the general effect of sweepback upon the variation of  $c_{m\delta}/\alpha_\delta$  with Mach number and indicate that the rate of reduction in  $c_{m\delta}/\alpha_\delta$  at a constant Mach number due to sweepback becomes greater as the angle of sweep is increased. The relationship between the curves is not clearly understood at this time; however, it appears that the sweep of the aileron hinge axis is a major controlling factor. It should be noted that the values of  $c_{m\delta}/\alpha_\delta$  given for the unswept wing represent the average obtained from the results presented in reference 2 for a 3-percent-thick and a 9-percent-thick airfoil section.

Figure 6 presents a comparison of the variation of the effective twisting-moment coefficients with Mach number as determined for different ailerons upon the same wing plan form.

Theoretically, if the values of  $\tau$  accurately weighed the effects of wing twist, wing geometry, and the aerodynamics of the wing-control configuration, the values of  $c_{m\delta}/\alpha_\delta$  for any given Mach number thus determined would be the same. That is, the curves in figures 5, 6, and 7 would be coincident. The fact that the values are not the same indicates that there are definite limitations in the applicability of the method at the present time. In the region between  $M \approx 0.8$  to  $M \approx 1.2$ , the values are quite different, but in the region between  $M \approx 1.2$  to  $M \approx 1.6$ , the values tend to agree more closely.

The major factor that can cause variations in the determination of  $c_{m\delta}/\alpha_\delta$  is the inability of the basic theory used to derive the weighing factor  $\tau$  to describe accurately the very complicated aerodynamic conditions existing in the transonic speed range.

Another possible contributing factor is the effect of wing bending caused by the differences in span loading due to control deflection and that due to damping. As was previously discussed under the subheading entitled "Résumé and Background," the variation in  $c_{m\delta}/\alpha_\delta$  due to

differential wing bending is negligible. However, for wing-control configurations similar to those tested, the previously discussed limitations should not apply, providing that the torsional stiffness criterion  $(\theta/m)_r$  is determined in the same manner.

Figure 7 presents the variation of effective twisting-moment coefficient with Mach number for an outboard 0.3-span, 0.25-chord aileron compared with a midspan spoiler of equal rigid-wing rolling effectiveness located upon the same wing at approximately the same chordwise station as the aileron hinge axis. The  $\tau$  values used for the spoiler were the same as for an aileron located at the same spanwise station. It is apparent that the twisting moment of the spoiler is much less than that of the aileron (approximately one-third); therefore, the merit of spoilers for control is very clearly illustrated where wing twisting is a problem.

The spoiler and aileron described were tested separately and in combination upon a wing which was constructed to have the values of the stiffness in torsion and bending scaled to that for a proposed fighter airplane in order that the effects of wing flexibility upon rolling effectiveness could be measured directly. The results shown in figure 8 serve to illustrate how the rolling effectiveness of a typical fighter airplane will be dependent upon the type of roll-control device selected. It should be noted that the fraction of rigid-wing rolling effectiveness retained by the spoiler is almost constant with increasing Mach number, whereas the aileron configuration exhibited severe loss of control effectiveness with increasing Mach number. The measured variation of  $\phi$  with Mach number for the configuration with the spoiler and aileron in combination is compared with that estimated from the data for the controls tested separately and the comparison shows that there was negligible interference between the spoiler and the aileron when operated in combination in this manner, thereby indicating that the values of  $c_{ms}/\alpha_8$  obtained for the controls tested separately could be used to predict the loss in rolling effectiveness for the controls in combination.

## CONCLUSIONS

A summary of some effective aerodynamic twisting-moment coefficients of various wing-control configurations at Mach numbers from 0.6 to 1.7 as determined by the use of rocket-propelled test vehicles indicates that, within the framework of the foregoing assumptions, the following conclusions may be drawn:

1. The value of the effective twisting-moment coefficient decreased as the sweepback of the aileron hinge axis is increased.

2. Large changes in the value of the effective twisting moment coefficient were obtained in the Mach number region from  $M \approx 0.8$  to  $M \approx 1.2$  with changes in aileron span and location upon the same wing plan form. From  $M \approx 1.2$  to  $M \approx 1.6$  the values tended to agree more closely.

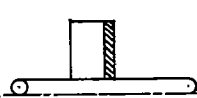
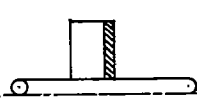
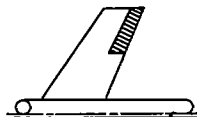
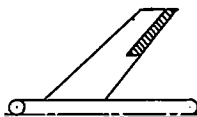
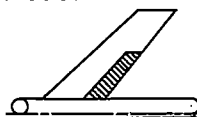
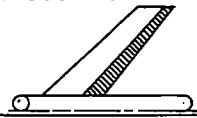
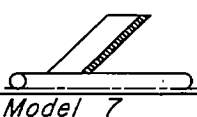
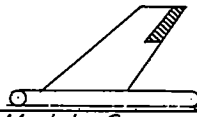

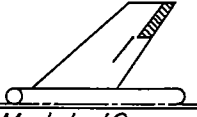
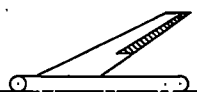
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*Table I*  
*Wing Control Configurations*

| Model 1<br>    | Wing Parameters |           |           |   | Control Parameters                   |           |       |            | Ref. |
|---|-----------------|-----------|-----------|---|--------------------------------------|-----------|-------|------------|------|
|   | A               | $\Lambda$ | $\lambda$ | Airfoil Section   | Type                                 | $c_{g/c}$ | $K_f$ | $K_\theta$ |      |
|                | 3.7             | 0         | 1.0       | NACA 65A003   | Aileron                              | 0.20      | 0.19  | 1.0        | 2    |
|   | 3.7             | 0         | 1.0       | NACA 65A009   | Aileron                              | .20       | .19   | 1.0        |      |
| Model 2<br>    | 4.0             | 35        | 0.60      | NACA 65A006   | Aileron                              | .30       | .57   | 1.0        | 3    |
| Model 3<br>    | 4.0             | 45        | .60       | NACA 65A006   | Aileron                              | .30       | .57   | 1.0        | 3    |
| Model 4<br>    | 4.0             | 45        | .60       | NACA 65A006   | Aileron                              | .30       | .14   | .57        | 3    |
| Model 5<br>   | 4.0             | 45        | .60       | NACA 65A006   | Aileron                              | .30       | .14   | 1.0        | 3    |
| Model 6<br>  | 3.7             | 45        | 1.0       | NACA 65A009   | Aileron                              | .20       | .19   | 1.0        | 2    |
| Model 7<br>  | 3.0             | 45        | .50       | NACA 0009-1/16<br>38/1/4(Modified) Root<br>NACA 0007-1/16<br>38/1/4(Modified) Tip | Aileron                              | .25       | .68   | 1.0        | 4    |
| Model 8<br>  | 3.0             | 45        | .50       | NACA 0009-1/16<br>38/1/4(Modified) Root<br>NACA 0007-1/16<br>38/1/4(Modified) Tip | Spoiler<br>$h/c = .06$<br>at<br>.75c | —         | .42   | .65        | 4    |
| Model 9<br>  | 3.0             | 45        | .50       | NACA 0009-1/16<br>38/1/4(Modified) Root<br>NACA 0007-1/16<br>38/1/4(Modified) Tip | Spoiler<br>and<br>Aileron            | —         | —     | —          | 4    |
| Model 10<br> | 3.5             | 61        | .25       | NACA 64A005   | Aileron                              | .30       | .50   | 1.0        | 5    |

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TABLE II  
VALUES OF THE AEROELASTIC WEIGHING FACTOR,  $\tau$

(a)  $A = 2$

| $K_0$                              | 0.4  | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4                                 | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4  | 0.5                                 | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   |  |  |
|------------------------------------|------|------|------|------|-------|-------|-------|-------------------------------------|------|------|------|-------|-------|-------|------|-------------------------------------|------|------|-------|-------|-------|--|--|
| $\lambda = 0; \Lambda = 0^\circ$   |      |      |      |      |       |       |       | $\lambda = 0; \Lambda = 40^\circ$   |      |      |      |       |       |       |      | $\lambda = 0; \Lambda = 60^\circ$   |      |      |       |       |       |  |  |
| 0                                  | 5.94 | 3.61 | 2.20 | 1.37 | 0.863 | 0.568 | 0.375 | 5.90                                | 3.51 | 2.12 | 1.33 | 0.830 | 0.559 | 0.375 | 5.70 | 3.41                                | 2.05 | 1.28 | 0.810 | 0.539 | 0.364 |  |  |
| .1                                 | 3.39 | 2.23 | 1.48 | .965 | .640  | .429  | .290  | 3.30                                | 2.18 | 1.43 | .935 | .610  | .420  | .290  | 3.22 | 2.11                                | 1.38 | .900 | .592  | .406  | .280  |  |  |
| .2                                 | 2.12 | 1.46 | 1.01 | .688 | .470  | .323  | .225  | 2.05                                | 1.44 | .990 | .670 | .450  | .318  | .225  | 2.00 | 1.38                                | .950 | .643 | .435  | .307  | .216  |  |  |
| .3                                 |      | 1.06 | .725 | .500 | .345  | .243  | .173  |                                     | 1.03 | .700 | .481 | .330  | .240  | .172  |      | 1.01                                | .690 | .470 | .322  | .231  | .167  |  |  |
| .4                                 |      |      | .536 | .370 | .254  | .176  | .134  |                                     |      | .560 | .355 | .245  | .179  | .133  |      |                                     | .515 | .349 | .236  | .174  | .128  |  |  |
| .5                                 |      |      |      | .272 | .187  | .139  | .103  |                                     |      |      | .272 | .179  | .136  | .105  |      |                                     |      | .258 | .175  | .132  | .100  |  |  |
| .6                                 |      |      |      |      | .138  | .104  | .080  |                                     |      |      |      | .133  | .103  | .079  |      |                                     |      |      | .128  | .100  | .077  |  |  |
| .7                                 |      |      |      |      |       | .079  | .061  |                                     |      |      |      |       | .079  | .061  |      |                                     |      |      |       | .077  | .060  |  |  |
| .8                                 |      |      |      |      |       |       | .048  |                                     |      |      |      |       |       | .047  |      |                                     |      |      |       |       | .047  |  |  |
| $\lambda = 0.5; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 0.5; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 0.5; \Lambda = 60^\circ$ |      |      |       |       |       |  |  |
| 0                                  | 3.80 | 2.81 | 2.09 | 1.52 | 1.10  | 0.870 | 0.690 | 3.65                                | 2.66 | 1.95 | 1.48 | 1.10  | 0.865 | 0.680 | 3.75 | 2.66                                | 1.86 | 1.41 | 1.08  | 0.848 | 0.670 |  |  |
| .1                                 | 2.20 | 1.68 | 1.31 | 1.03 | .820  | .655  | .535  | 2.15                                | 1.65 | 1.29 | 1.03 | .810  | .658  | .530  | 2.20 | 1.67                                | 1.25 | .998 | .785  | .641  | .525  |  |  |
| .2                                 | 1.50 | 1.18 | .970 | .775 | .630  | .516  | .435  | 1.43                                | 1.16 | .940 | .775 | .630  | .524  | .435  | 1.47 | 1.16                                | .930 | .755 | .610  | .516  | .430  |  |  |
| .3                                 |      | .940 | .760 | .619 | .505  | .425  | .365  |                                     | .910 | .730 | .606 | .505  | .428  | .363  |      | .918                                | .720 | .593 | .490  | .420  | .360  |  |  |
| .4                                 |      |      | .625 | .507 | .420  | .354  | .320  |                                     |      | .590 | .492 | .415  | .357  | .310  |      |                                     | .580 | .483 | .410  | .350  | .310  |  |  |
| .5                                 |      |      |      | .429 | .350  | .301  | .270  |                                     |      |      | .416 | .395  | .303  | .265  |      |                                     |      | .403 | .380  | .299  | .265  |  |  |
| .6                                 |      |      |      |      | .295  | .261  | .235  |                                     |      |      |      | .293  | .262  | .231  |      |                                     |      |      | .285  | .259  | .232  |  |  |
| .7                                 |      |      |      |      |       | .231  | .205  |                                     |      |      |      |       | .230  | .205  |      |                                     |      |      |       | .228  | .204  |  |  |
| .8                                 |      |      |      |      |       |       | .182  |                                     |      |      |      |       |       | .182  |      |                                     |      |      |       |       | .180  |  |  |
| $\lambda = 1.0; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 1.0; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 1.0; \Lambda = 60^\circ$ |      |      |       |       |       |  |  |
| 0                                  | 1.59 | 1.64 | 1.75 | 1.46 | 1.24  | 1.10  | 0.950 | 1.55                                | 1.63 | 1.70 | 1.45 | 1.25  | 1.09  | 0.960 | 1.54 | 1.61                                | 1.68 | 1.42 | 1.23  | 1.08  | 0.950 |  |  |
| .1                                 | .97  | 1.05 | 1.14 | 1.00 | .880  | .788  | .700  | .950                                | 1.03 | 1.12 | 1.00 | .890  | .800  | .715  | .950 | 1.03                                | 1.11 | .984 | .870  | .783  | .710  |  |  |
| .2                                 | .66  | .746 | .840 | .770 | .702  | .630  | .572  | .645                                | .715 | .830 | .760 | .710  | .643  | .590  | .645 | .724                                | .825 | .750 | .695  | .640  | .590  |  |  |
| .3                                 |      | .645 | .705 | .640 | .585  | .535  | .490  |                                     | .640 | .690 | .640 | .591  | .544  | .502  |      | .630                                | .680 | .630 | .592  | .540  | .506  |  |  |
| .4                                 |      |      | .630 | .566 | .505  | .470  | .440  |                                     |      | .625 | .585 | .510  | .476  | .450  |      |                                     | .620 | .555 | .510  | .476  | .450  |  |  |
| .5                                 |      |      |      | .500 | .442  | .422  | .400  |                                     |      |      | .500 | .450  | .430  | .410  |      |                                     |      | .500 | .445  | .427  | .410  |  |  |
| .6                                 |      |      |      |      | .398  | .381  | .373  |                                     |      |      |      | .403  | .390  | .380  |      |                                     |      |      | .400  | .390  | .379  |  |  |
| .7                                 |      |      |      |      |       | .365  | .350  |                                     |      |      |      |       | .363  | .355  |      |                                     |      |      |       | .370  | .355  |  |  |
| .8                                 |      |      |      |      |       |       | .335  |                                     |      |      |      |       |       | .340  |      |                                     |      |      |       |       | .340  |  |  |

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TABLE II

VALUES OF THE AEROELASTIC WEIGHING FACTOR,  $\tau$  - Continued(b)  $A = 4$ 

| $K_0$<br>$K_1$                     | 0.4  | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4                                 | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4  | 0.5                                 | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   |  |  |
|------------------------------------|------|------|------|------|-------|-------|-------|-------------------------------------|------|------|------|-------|-------|-------|------|-------------------------------------|------|------|-------|-------|-------|--|--|
| $\lambda = 0; \Lambda = 0^\circ$   |      |      |      |      |       |       |       | $\lambda = 0; \Lambda = 40^\circ$   |      |      |      |       |       |       |      | $\lambda = 0; \Lambda = 60^\circ$   |      |      |       |       |       |  |  |
| 0                                  | 5.33 | 3.28 | 2.03 | 1.28 | 0.810 | 0.540 | 0.361 | 5.04                                | 3.20 | 1.92 | 1.20 | 0.755 | 0.518 | 0.355 | 4.35 | 2.71                                | 1.72 | 1.10 | 0.710 | 0.492 | 0.380 |  |  |
| .1                                 | 2.95 | 1.97 | 1.32 | .868 | .580  | .400  | .279  | 2.75                                | 1.97 | 1.24 | .820 | .545  | .382  | .274  | 2.40 | 1.63                                | 1.12 | .758 | .510  | .368  | .263  |  |  |
| .2                                 | 1.87 | 1.31 | .930 | .630 | .430  | .308  | .218  | 1.76                                | 1.23 | .870 | .598 | .409  | .294  | .214  | 1.55 | 1.10                                | .790 | .550 | .386  | .281  | .208  |  |  |
| .3                                 |      | .975 | .685 | .479 | .365  | .242  | .175  |                                     | .920 | .645 | .450 | .315  | .232  | .173  |      | .828                                | .585 | .418 | .300  | .222  | .165  |  |  |
| .4                                 |      |      | .513 | .365 | .260  | .192  | .143  |                                     |      | .490 | .346 | .291  | .184  | .139  |      |                                     | .450 | .323 | .235  | .188  | .135  |  |  |
| .5                                 |      |      |      | .292 | .208  | .156  | .117  |                                     |      |      | .278 | .195  | .149  | .113  |      |                                     |      | .259 | .187  | .143  | .109  |  |  |
| .6                                 |      |      |      |      | .167  | .126  | .096  |                                     |      |      |      | .157  | .121  | .093  |      |                                     |      |      | .150  | .127  | .090  |  |  |
| .7                                 |      |      |      |      |       | .105  | .079  |                                     |      |      |      |       | .100  | .077  |      |                                     |      |      | .960  | .075  | .062  |  |  |
| .8                                 |      |      |      |      |       |       | .066  |                                     |      |      |      |       |       | .064  |      |                                     |      |      |       |       | .062  |  |  |
| $\lambda = 0.5; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 0.5; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 0.5; \Lambda = 60^\circ$ |      |      |       |       |       |  |  |
| 0                                  | 3.99 | 2.86 | 2.08 | 1.55 | 1.15  | 0.900 | 0.708 | 3.84                                | 2.80 | 2.00 | 1.49 | 1.08  | 0.860 | 0.685 | 3.60 | 2.56                                | 1.83 | 1.39 | 1.06  | 0.835 | 0.665 |  |  |
| .1                                 | 2.25 | 1.74 | 1.35 | 1.06 | .840  | .685  | .555  | 2.15                                | 1.68 | 1.30 | 1.01 | .790  | .645  | .525  | 1.97 | 1.51                                | 1.16 | .930 | .750  | .615  | .504  |  |  |
| .2                                 | 1.56 | 1.23 | 1.00 | .803 | .650  | .540  | .450  | 1.47                                | 1.20 | .950 | .760 | .610  | .505  | .425  | 1.32 | 1.06                                | .850 | .703 | .583  | .486  | .406  |  |  |
| .3                                 |      | .980 | .765 | .636 | .528  | .450  | .385  |                                     | .910 | .730 | .600 | .500  | .422  | .358  |      | .840                                | .665 | .560 | .474  | .400  | .385  |  |  |
| .4                                 |      |      | .610 | .522 | .440  | .381  | .330  |                                     |      | .580 | .490 | .420  | .356  | .311  |      |                                     | .535 | .460 | .400  | .342  | .295  |  |  |
| .5                                 |      |      |      | .445 | .374  | .327  | .288  |                                     |      |      | .421 | .355  | .311  | .273  |      |                                     |      | .398 | .340  | .298  | .260  |  |  |
| .6                                 |      |      |      |      | .323  | .288  | .254  |                                     |      |      |      | .307  | .273  | .243  |      |                                     |      |      | .296  | .261  | .232  |  |  |
| .7                                 |      |      |      |      |       | .257  | .227  |                                     |      |      |      |       | .248  | .220  |      |                                     |      |      |       | .236  | .208  |  |  |
| .8                                 |      |      |      |      |       |       | .204  |                                     |      |      |      |       |       | .200  |      |                                     |      |      |       |       | .189  |  |  |
| $\lambda = 1.0; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 1.0; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 1.0; \Lambda = 60^\circ$ |      |      |       |       |       |  |  |
| 0                                  | 1.68 | 1.73 | 1.78 | 1.51 | 1.29  | 1.12  | 0.972 | 1.62                                | 1.68 | 1.78 | 1.46 | 1.24  | 1.06  | 0.910 | 1.53 | 1.54                                | 1.56 | 1.36 | 1.21  | 1.04  | 0.890 |  |  |
| .1                                 | .990 | 1.04 | 1.11 | .99  | .890  | .800  | .725  | .952                                | 1.01 | 1.07 | .953 | .860  | .770  | .690  | .930 | .960                                | .980 | .910 | .840  | .754  | .673  |  |  |
| .2                                 | .690 | .770 | .860 | .770 | .690  | .640  | .592  | .665                                | .744 | .840 | .750 | .680  | .620  | .565  | .670 | .671                                | .765 | .710 | .670  | .602  | .555  |  |  |
| .3                                 |      | .660 | .720 | .642 | .578  | .540  | .501  |                                     | .634 | .695 | .626 | .566  | .522  | .485  |      | .603                                | .635 | .596 | .556  | .514  | .473  |  |  |
| .4                                 |      |      | .626 | .560 | .500  | .470  | .441  |                                     |      | .605 | .540 | .490  | .460  | .430  |      |                                     | .550 | .515 | .485  | .450  | .420  |  |  |
| .5                                 |      |      |      | .496 | .442  | .420  | .399  |                                     |      |      | .489 | .437  | .420  | .385  |      |                                     |      | .460 | .433  | .402  | .378  |  |  |
| .6                                 |      |      |      |      | .400  | .380  | .364  |                                     |      |      |      | .396  | .375  | .353  |      |                                     |      |      | .390  | .365  | .346  |  |  |
| .7                                 |      |      |      |      |       | .352  | .338  |                                     |      |      |      |       | .349  | .325  |      |                                     |      |      |       | .341  | .321  |  |  |
| .8                                 |      |      |      |      |       |       | .314  |                                     |      |      |      |       |       | .305  |      |                                     |      |      |       |       | .300  |  |  |

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TABLE II  
VALUES OF THE AEROELASTIC WEIGHTING FACTOR,  $\tau$  - Concluded

(c)  $A = 8$ 

| $K_0$<br>$K_1$                     | 0.4  | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4                                 | 0.5  | 0.6  | 0.7  | 0.8   | 0.9   | 1.0   | 0.4  | 0.5                                 | 0.6  | 0.7   | 0.8   | 0.9   | 1.0   |  |  |
|------------------------------------|------|------|------|------|-------|-------|-------|-------------------------------------|------|------|------|-------|-------|-------|------|-------------------------------------|------|-------|-------|-------|-------|--|--|
| $\lambda = 0; \Lambda = 0^\circ$   |      |      |      |      |       |       |       | $\lambda = 0; \Lambda = 40^\circ$   |      |      |      |       |       |       |      | $\lambda = 0; \Lambda = 60^\circ$   |      |       |       |       |       |  |  |
| 0                                  | 5.00 | 3.08 | 1.91 | 1.22 | 0.775 | 0.555 | 0.400 | 4.30                                | 2.67 | 1.67 | 1.07 | 0.698 | 0.482 | 0.336 | 3.42 | 2.20                                | 1.41 | 0.940 | 0.630 | 0.440 | 0.315 |  |  |
| .1                                 | 2.71 | 1.81 | 1.22 | .823 | .555  | .396  | .282  | 2.32                                | 1.57 | 1.07 | .725 | .490  | .352  | .255  | 1.83 | 1.27                                | .880 | .630  | .447  | .328  |       |  |  |
| .2                                 | 1.71 | 1.21 | .855 | .597 | .418  | .298  | .212  | 1.46                                | 1.05 | .765 | .536 | .375  | .273  | .201  | 1.15 | .862                                | .647 | .473  | .385  | .206  |       |  |  |
| .3                                 |      | .910 | .640 | .452 | .340  | .230  | .167  |                                     | .800 | .570 | .407 | .292  | .216  | .162  |      | .688                                | .510 | .375  | .275  | .205  |       |  |  |
| .4                                 |      |      | .490 | .352 | .252  | .185  | .135  |                                     |      | .438 | .320 | .235  | .177  | .134  |      |                                     | .420 | .310  | .257  | .170  |       |  |  |
| .5                                 |      |      |      | .284 | .203  | .150  | .111  |                                     |      |      | .265 | .194  | .145  | .109  |      |                                     |      | .259  | .191  | .140  |       |  |  |
| .6                                 |      |      |      |      | .165  | .124  | .092  |                                     |      |      | .163 | .121  | .095  | .076  |      |                                     |      |       | .163  | .119  |       |  |  |
| .7                                 |      |      |      |      |       | .115  | .077  |                                     |      |      |      | .103  | .076  | .064  |      |                                     |      |       |       | .101  |       |  |  |
| .8                                 |      |      |      |      |       |       | .065  |                                     |      |      |      |       |       |       |      |                                     |      |       |       | .063  |       |  |  |
| $\lambda = 0.5; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 0.5; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 0.5; \Lambda = 60^\circ$ |      |       |       |       |       |  |  |
| 0                                  | 4.20 | 3.00 | 2.15 | 1.60 | 1.19  | 0.915 | 0.705 | 3.60                                | 2.55 | 1.84 | 1.40 | 1.08  | 0.848 | 0.670 | 3.20 | 2.31                                | 1.68 | 1.30  | 1.01  | 0.810 | 0.650 |  |  |
| .1                                 | 2.30 | 1.77 | 1.35 | 1.05 | .820  | .682  | .535  | 2.05                                | 1.54 | 1.17 | .943 | .775  | .630  | .508  | 1.83 | 1.41                                | 1.08 | .890  | .735  | .600  |       |  |  |
| .2                                 | 1.52 | 1.21 | .980 | .781 | .635  | .520  | .427  | 1.34                                | 1.08 | .860 | .728 | .600  | .499  | .410  | 1.21 | .980                                | .800 | .680  | .575  | .474  |       |  |  |
| .3                                 |      | .943 | .747 | .620 | .512  | .426  | .356  |                                     | .860 | .680 | .580 | .490  | .410  | .342  |      | .780                                | .630 | .545  | .473  | .396  |       |  |  |
| .4                                 |      |      | .595 | .500 | .425  | .358  | .305  |                                     |      | .555 | .480 | .410  | .349  | .295  |      |                                     | .505 | .450  | .397  | .337  |       |  |  |
| .5                                 |      |      |      | .388 | .362  | .310  | .265  |                                     |      |      | .410 | .352  | .300  | .257  |      |                                     |      | .389  | .340  | .292  |       |  |  |
| .6                                 |      |      |      |      | .310  | .270  | .233  |                                     |      |      |      | .305  | .262  | .226  |      |                                     |      |       | .298  | .257  |       |  |  |
| .7                                 |      |      |      |      |       | .241  | .208  |                                     |      |      |      |       | .235  | .203  |      |                                     |      |       |       | .230  |       |  |  |
| .8                                 |      |      |      |      |       |       | .186  |                                     |      |      |      |       |       | .182  |      |                                     |      |       |       | .179  |       |  |  |
| $\lambda = 1.0; \Lambda = 0^\circ$ |      |      |      |      |       |       |       | $\lambda = 1.0; \Lambda = 40^\circ$ |      |      |      |       |       |       |      | $\lambda = 1.0; \Lambda = 60^\circ$ |      |       |       |       |       |  |  |
| 0                                  | 1.78 | 1.86 | 1.89 | 1.59 | 1.35  | 1.17  | 1.02  | 1.58                                | 1.61 | 1.66 | 1.43 | 1.24  | 1.07  | 0.940 | 1.58 | 1.50                                | 1.43 | 1.31  | 1.22  | 1.06  | 0.930 |  |  |
| .1                                 | 1.03 | 1.10 | 1.17 | 1.04 | .927  | .833  | .746  | 1.01                                | 1.03 | 1.04 | .950 | .870  | .790  | .710  | 1.00 | .960                                | .910 | .880  | .850  | .772  |       |  |  |
| .2                                 | .740 | .822 | .910 | .820 | .730  | .665  | .610  | .740                                | .770 | .806 | .745 | .690  | .633  | .581  | .741 | .740                                | .740 | .703  | .670  | .620  |       |  |  |
| .3                                 |      | .692 | .780 | .675 | .600  | .556  | .515  |                                     | .608 | .675 | .625 | .576  | .535  | .500  |      | .651                                | .640 | .600  | .565  | .525  |       |  |  |
| .4                                 |      |      | .632 | .580 | .514  | .482  | .450  |                                     |      | .584 | .543 | .500  | .470  | .440  |      |                                     | .578 | .530  | .488  | .456  |       |  |  |
| .5                                 |      |      |      | .515 | .454  | .430  | .408  |                                     |      |      | .480 | .444  | .421  | .398  |      |                                     |      | .474  | .433  | .408  |       |  |  |
| .6                                 |      |      |      |      | .405  | .388  | .371  |                                     |      |      |      | .400  | .380  | .364  |      |                                     |      |       | .391  | .372  |       |  |  |
| .7                                 |      |      |      |      |       | .360  | .345  |                                     |      |      |      |       | .354  | .335  |      |                                     |      |       |       | .346  |       |  |  |
| .8                                 |      |      |      |      |       |       | .320  |                                     |      |      |      |       |       | .315  |      |                                     |      |       |       | .308  |       |  |  |

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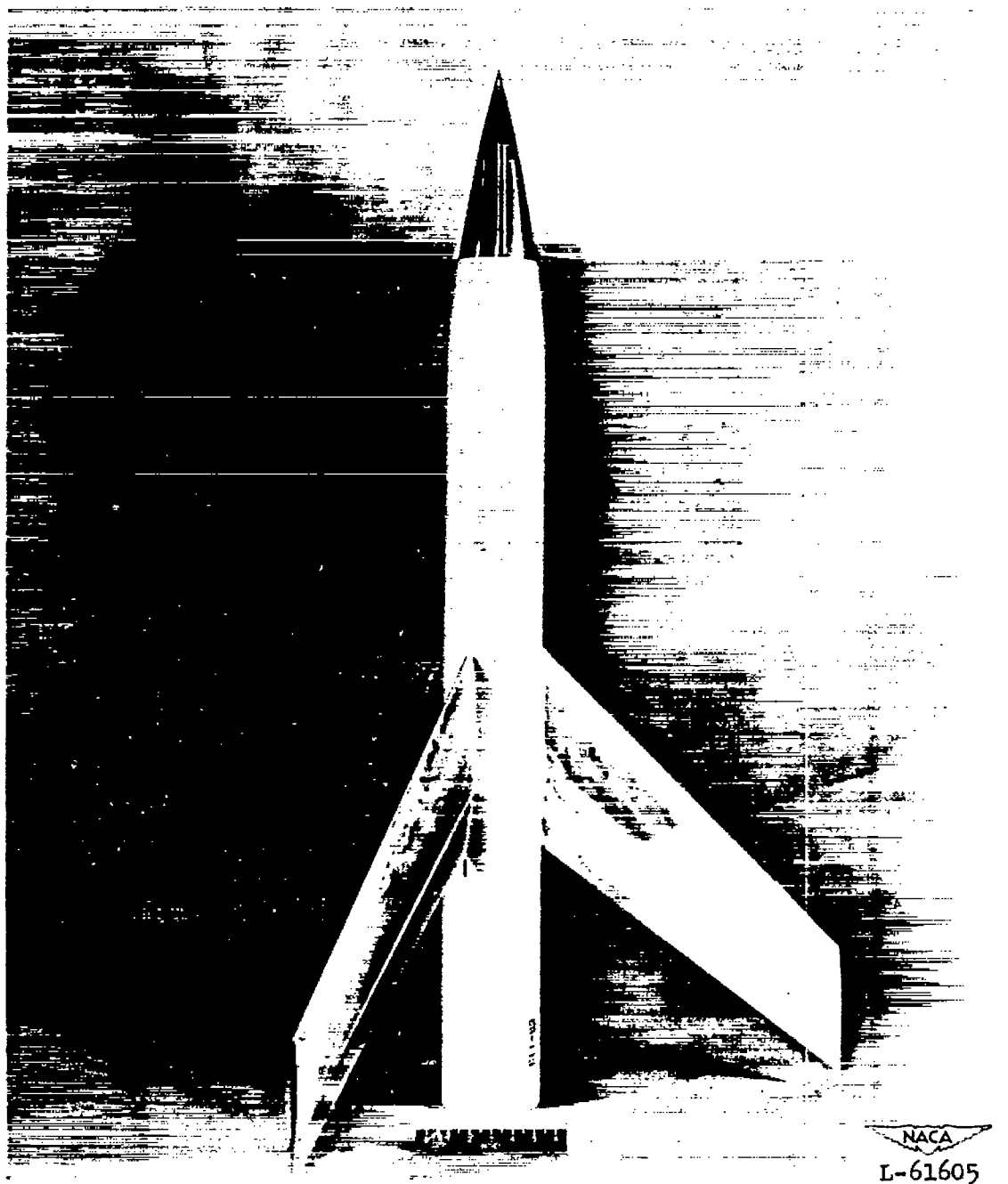
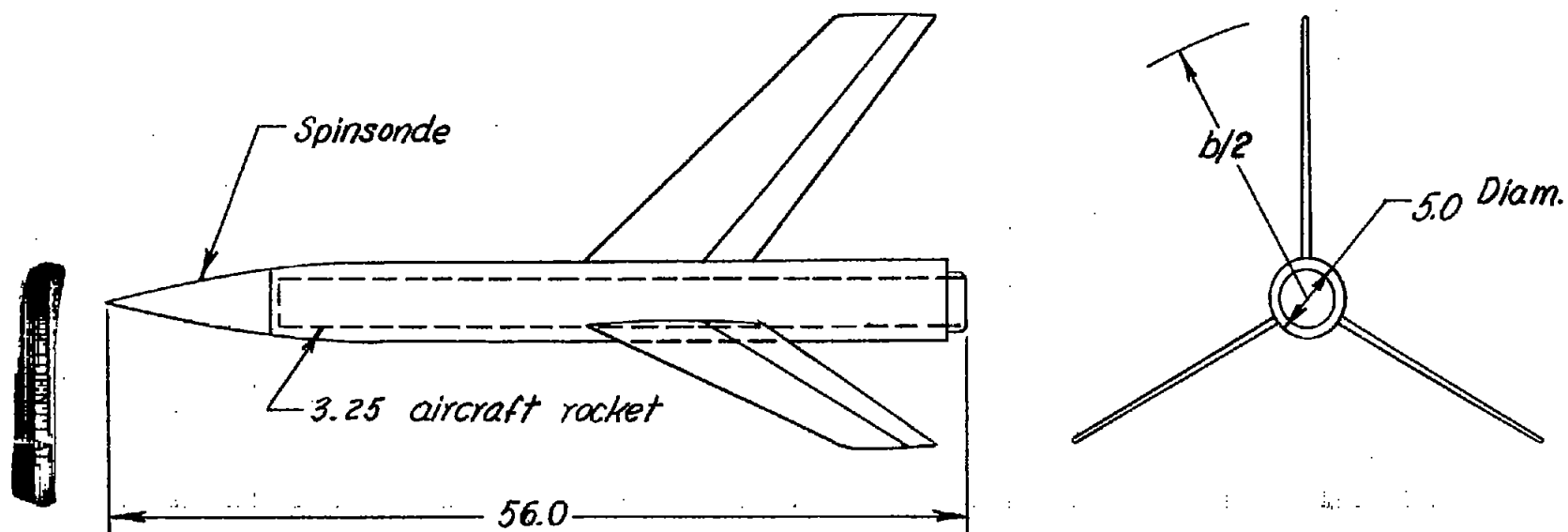


Figure 1.- Typical test vehicle.



*All dimensions in inches*

Figure 2.- General arrangement of typical test vehicle.



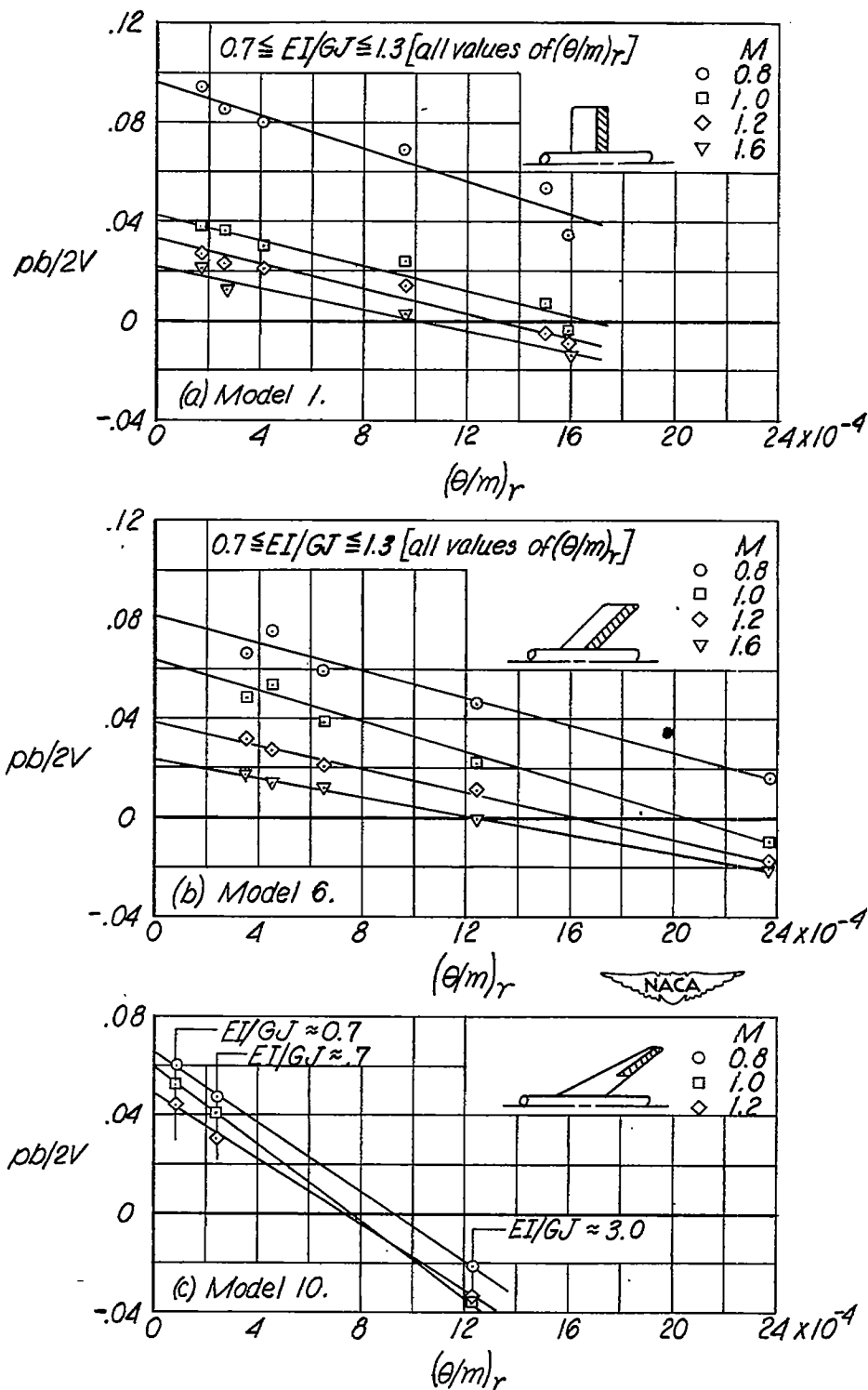


Figure 3.- Typical variation of rolling effectiveness with the torsional stiffness parameter.

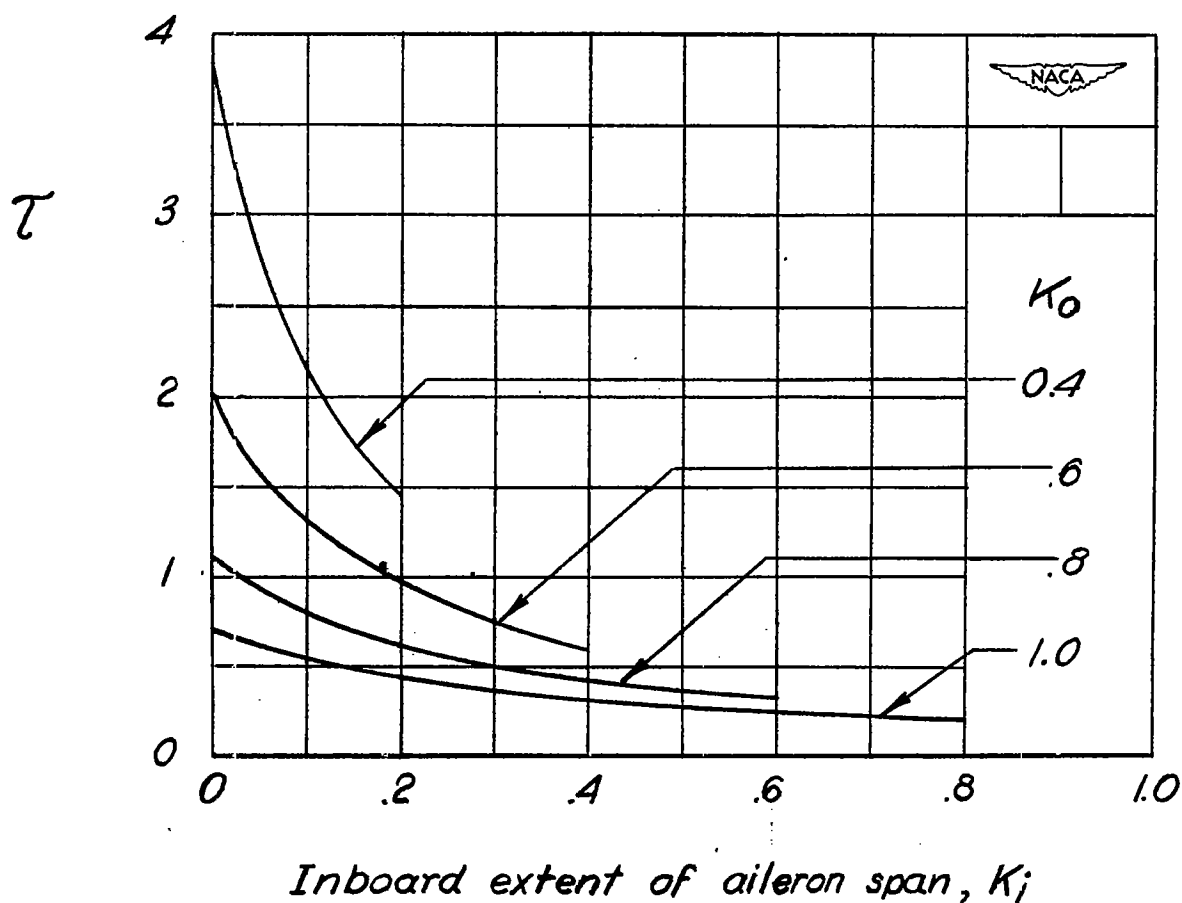


Figure 4.- Typical effect of aileron span and spanwise location upon the aeroelastic weighing factor  $\tau$ .  $A = 4$ ;  $\lambda = 0.5$ ;  $\Lambda = 40^\circ$ .

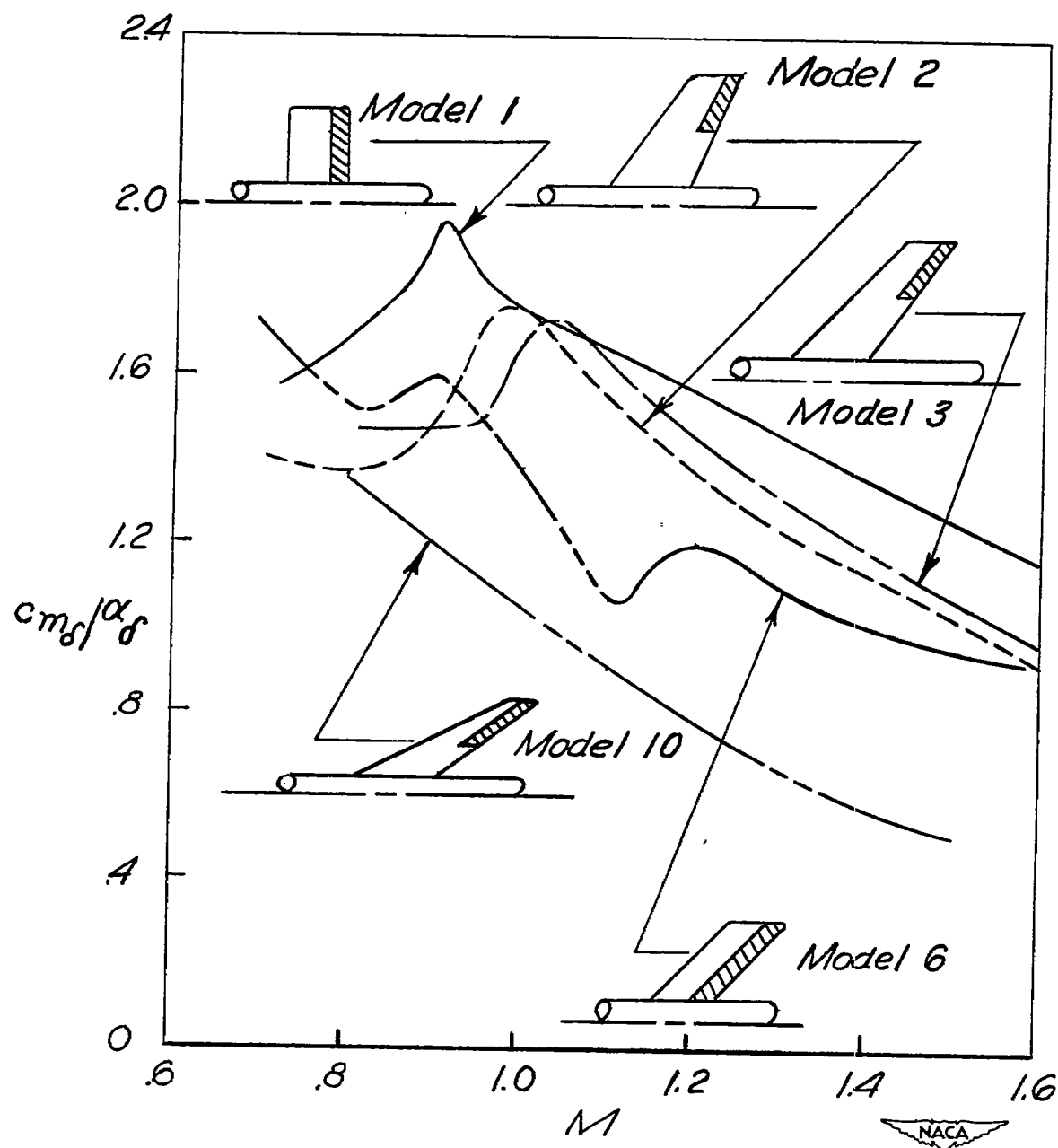


Figure 5.- Effect of sweepback upon the variation of the effective twisting-moment coefficient with Mach number.

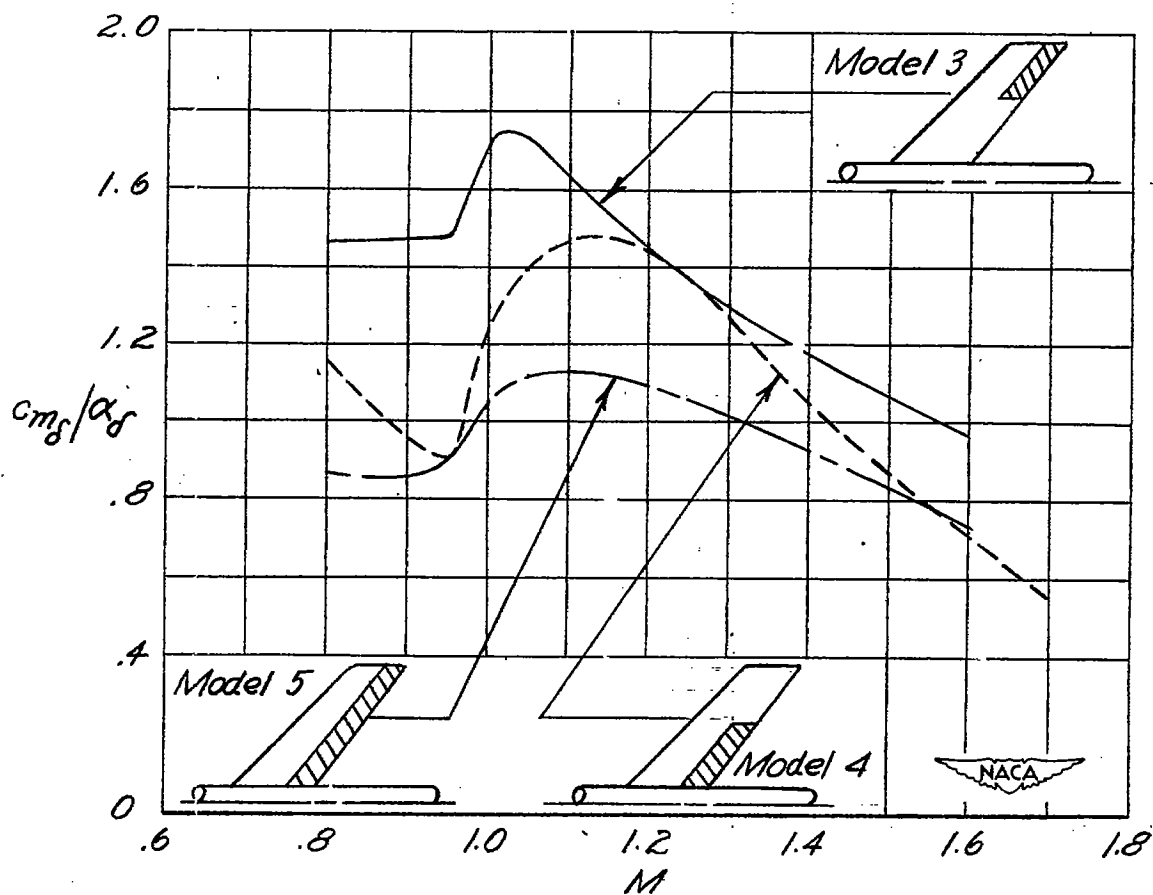


Figure 6.- Variation of effective twisting-moment coefficient with Mach number for different ailerons upon the same wing plan form.

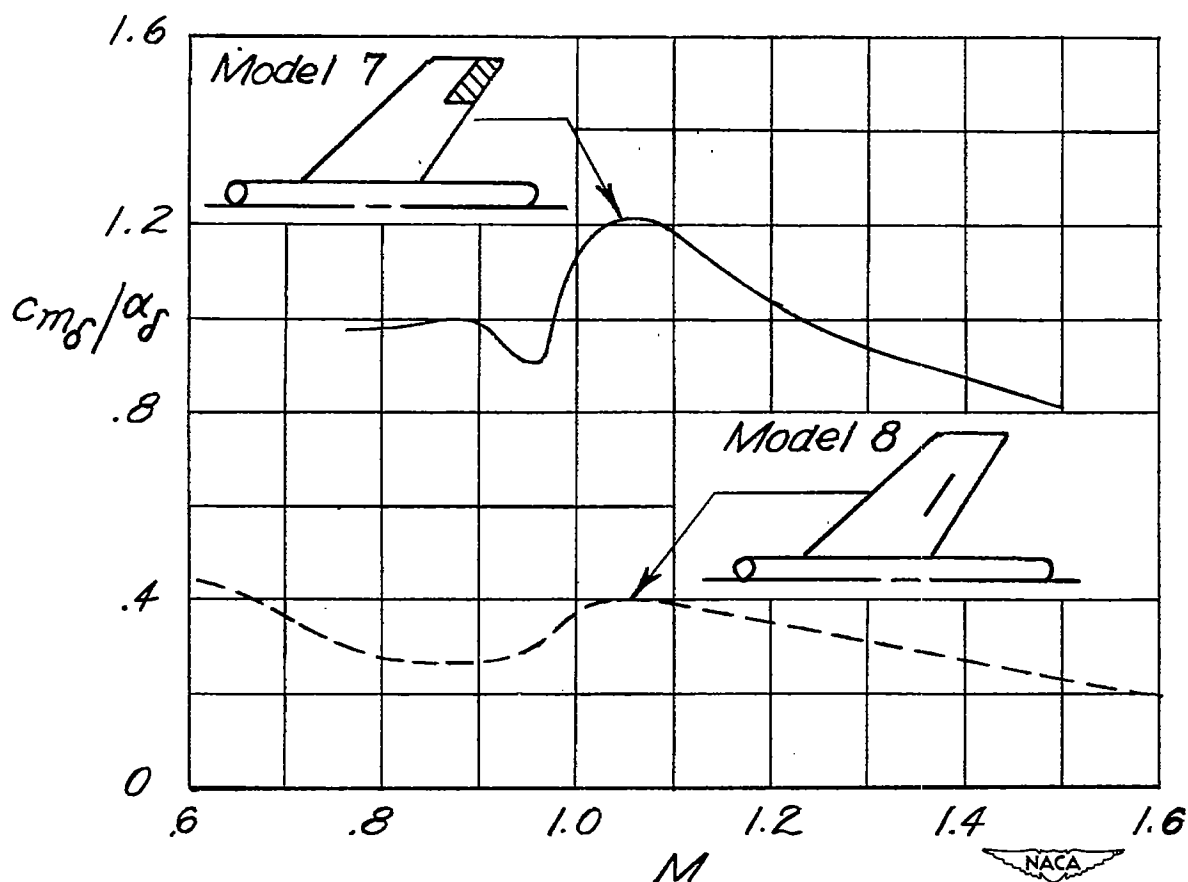


Figure 7.- Variation of effective twisting-moment coefficient with Mach number for an outboard 0.3-span, 0.25-chord aileron and for a midspan spoiler.

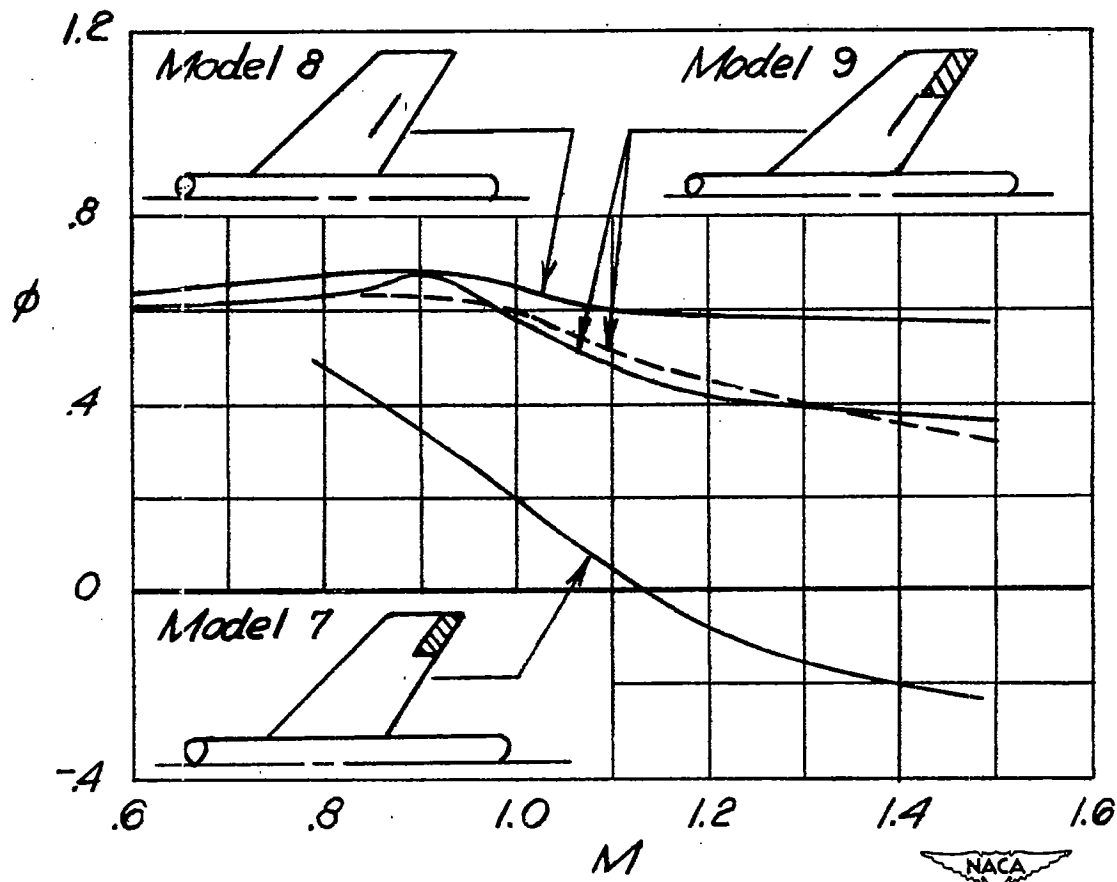


Figure 8.- Effect of type of roll-control device upon fraction of rigid-wing rolling effectiveness retained by a wing constructed to have stiffness in torsion and bending scaled to that for a proposed fighter airplane.